18103098 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.) EXAMINATION, JUNE 2018

Second Semester

Core Course—MM2CRT01—ANALYTIC GEOMETRY, TRIGONOMETRY AND DIFFERENTIAL CALCULUS

[For Mathematics M I, Mathematics M II Computer Science and Computer Application M III (Triple Main) Programmes]

(2017 Admissions only)

Time : Three Hours Maximum : 80 Marks

Part A

Answer any **ten** *questions.*
Each question carries 2 *marks.*

1. Write the equation of tangent to the parabola $y^2 = 6x$ at $(6, 6)$.

2. Find the pole of the line $x + y + 1 = 0$ with respect to the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

3. Write the equation of the normal to $\frac{x^2}{4} - \frac{y^2}{9} = 1$ at $\left(2\sqrt{2}, 3\right)$.

4. Find the orthoptic locus of the parabola $y^2 = 4x$.

5. Derive the polar equation of a straight line.

6. Find the locus of the foot of the perpendiculars drawn from the pole to the tangents to the circle $r = 2a\cos\theta$.

7. Prove that $\cosh^2 y - \sinh^2 y = 1$.

8. Prove that $\tanh 3x = \frac{3\tanh x + \tanh^3 x}{1 + 3\tanh^2 x}$.

9. Separate into real and imaginary parts : $f(z) = \sin z$.

Turn over

10. If $y = \sin(\sin x)$, find $\frac{dy}{dx}$.

11. Find the n^{th} derivative of $\log(3x+5)$.

12. Evaluate $\lim_{x \to 0}(x \log x)$.

(10 × 2 = 20 marks)

Part B

Answer any **six** *questions.*
Each question carries 5 *marks.*

13. Is SY and SY′ be perpendicular from the foci upon the tangent at a point P of the ellipse. Prove that Y and Y′ lie on a circle.

14. Find the locus of the poles of chords of the parabola $y^2 = 4ax$ subtending a right angle at the vertex.

15. Derive the polar equation of a conic.

16. Prove that $\log\left(\frac{a+ib}{a-ib}\right) = 2i \tan^{-1}\left(\frac{b}{a}\right)$.

17. Factorize $x^5 - 1$.

18. Factorize $x^{10} - 2x^5 \cos\pi/3 + 1$.

19. If $x = \sin t$, $y = \sin t$, prove that, $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2 y = 0$.

20. Find the n^{th} derivative of $\frac{x+1}{x^2-4}$.

21. If $y = a\cos\log x + b\sin\log x$, prove that $x^2 y_{n+2} + (2n+1)x\, y_{n+1} + \left(n^2+1\right) y_n = 0$.

(6 × 5 = 30 marks)

Part C

Answer any **two** *questions.*
Each question carries 15 *marks.*

22. (a) The polar of a point P with respect to the parabola $y^2 = 4ax$ meets the curve in Q and R. Show that if P lies on the line $lx + my + n = 0$, then the middle point of QR lies on the parabola $l\left(y^2 - 4ax\right) + 2a\left(lx + my + n\right) = 0$.

(b) Prove that the eccentric angles of the ends of a pair of conjugate diameters differ by a right angle.

23. (a) A chord PQ of a conic subtends an angle 2α of constant magnitude at the pole. Find the locus of the intersection of the tangents at P and Q.

(b) If the focal chord of an ellipse makes an angle α with the major axis, show that the angle between the tangents at the extremities is $\tan^{-1}\left(\frac{2e\sin\alpha}{1-e^2}\right)$, where e is the eccentricity.

24. (a) Sum the series $\frac{1}{2}\sin\alpha + \frac{1.3}{2.4}\sin 2\alpha + \frac{1.3.5}{2.4.6}\sin 3\alpha + \ldots$. to infinity.

(b) Factorize $x^6 + 1$.

25. (a) If $y = \sin\left(m\sin^{-1}x\right)$, show that $\left(1-x^2\right)y_{n+2} = (2n+1)\,x\,y_{n+1} + \left(n^2 - m^2\right)y_n$.

(b) Evaluate $\lim_{x\to\frac{\pi}{2}} (\sin x)^{\tan x}$.

(2 × 15 = 30 marks)